We are now in a position to determine the volume of fluid V_o that must exit the compression chamber via the knock-off tube in order for the pressure within the compression chamber to decay to ambient conditions in accordance with the postulated pressure-time function given by (4). We write

$$V_o = \int_0^\infty Q\, dt \tag{51}$$

Since the volume rate of flow Q diminished in an exponential fashion, the limits of integration on the time t are zero and infinity.

From the combination of equations (50) and (51), we have

$$V_o = \frac{4\pi P_g}{\rho L} \sum_{j=1}^{\infty} \left[\frac{1}{\frac{1}{\Theta_1} - \nu\lambda_j^2}\right] \left(\frac{1}{\lambda_j^2}\right) \int_0^\infty \left[e^{-\lambda_j^2 \nu t} - e^{-t/\Theta_1}\right] dt \tag{52}$$

from which

$$V_o = \frac{4\pi P_g \Theta_1}{\mu L} \sum_{j=1}^{\infty} \frac{1}{\lambda_j^4} \tag{53}$$

Recalling that λ_j is defined by equation (45), we can obtain from reference (e)

$$\begin{aligned} \lambda_1 R_o &= 2.4048 \\ \lambda_2 R_o &= 5.5201 \\ \lambda_3 R_o &= 8.6537 \\ \lambda_4 R_o &= 11.7915 \end{aligned} \tag{54}$$

etc.

Thus equation (53) becomes

$$V_o \approx \frac{\pi \Theta_1 P_g R_o^4}{8 \mu L} \tag{55}$$

The time constant Θ_1 can be found from equation (55) providing V_o is known. It is understood that the pressure in the compression chamber is increased by forcing an additional quantity of fluid into the chamber. From reference (f) the compressibility of the subject fluids (SAE 10 and SAE 20 oil) can be closely approximated from the following equation

$$\frac{V_1}{V_2} = 1.00 - (4.31 \times 10^{-6}) P_g + (6.51 \times 10^{-11}) P_g^2 - (5.03 \times 10^{-16}) P_g^3 \tag{56}$$

where the nomenclature is

V_1 . . . volume of fluid under pressure P_g (volume of compression chamber), in^3

V_2 . . . volume of fluid under atmospheric pressure (volume that would create pressure P_g if compressed to volume V_1), in^3

Since $V_o = V_2 - V_1$, then

$$V_o = V_1 P_g \left[\frac{(4.31 \times 10^{-6}) - (6.51 \times 10^{-11}) P_g + (5.03 \times 10^{-16}) P_g^2}{1.00 - (4.31 \times 10^{-6}) P_g + (6.51 \times 10^{-11}) P_g^2 - (5.03 \times 10^{-16}) P_g^3} \right] \tag{57}$$

To simplify the writing of (57), we introduce the term K_p, defined as follows for the subject fluids